Stress tensor distribution around static quark and anti-quark in SU(3) Yang-Mills theory

Ryosuke Yanagihara (Osaka University)

Special thanks to
Takumi Iritani (RIKEN), Masakiyo Kitazawa (Osaka), Masayuki Asakawa (Osaka), Tetsuo Hatsuda (RIKEN)
QED vs. QCD

QED
- Electric fields spread all over the space
- Coulomb potential

QCD
- Flux tube, squeezed one-dimensionally
- Confinement potential
QED vs. QCD

QED
- Electric fields spread all over the space
- Coulomb potential

QCD
- Flux tube, squeezed one-dimensionally
- Confinement potential

Local Interaction

Maxwell stress
Energy Momentum Tensor (EMT)

\[ T_{\mu\nu} = \begin{pmatrix} T_{00} & T_{01} & T_{02} & T_{03} \\ T_{10} & T_{11} & T_{12} & T_{13} \\ T_{20} & T_{21} & T_{22} & T_{23} \\ T_{30} & T_{31} & T_{32} & T_{33} \end{pmatrix} \]

- Energy density
- Stress tensor
- Momentum density
- Pressure

** ✓ Stress is force per unit area**

\[ f_i = \sigma_{ij} n_j ; \quad \sigma_{ij} = -T_{ij} \]

Landau and Lifshitz
Energy Momentum Tensor (EMT)

Stress is force per unit area:

\[ f_i = \sigma_{ij} n_j ; \quad \sigma_{ij} = -T_{ij} \]

Landau and Lifshitz
Maxwell Stress

\[ T_{ij} = \epsilon_0 \left( E_i E_j - \frac{\delta_{ij}}{2} E^2 \right) + \frac{1}{\mu_0} \left( B_i B_j - \frac{\delta_{ij}}{2} B^2 \right) \]

Perpendicular plane: \( \lambda_k < 0 \)
Parallel plane: \( \lambda_k > 0 \)

Stress tensor
\[ T_{ij} n_j^{(k)} = \lambda_k n_i^{(k)} \]
\[ (i, j = 1, 2, 3 ; k = 1, 2, 3) \]

Length of arrows = \( \sqrt{|\lambda_k|} \)
QED vs. QCD

- QED
  - Electric fields spread all over the space
  - Coulomb potential

- QCD
  - Flux tube, squeezed one-dimensionally
  - Confinement potential

Maxwell stress

Local Interaction
Goal and Method

Stress distribution in $Q\bar{Q}$ system in terms of local interaction

Gradient Flow

Lattice study

RY+ (FlowQCD collab.), arXiv:1803.05656

Model Analysis

RY, Takumi Iritani and Masakiyo Kitazawa
in progress

Dual Abelian-Higgs model
Stress tensor distribution around static $Q\bar{Q}$ via Yang–Mills gradient flow

RY+ (FlowQCD collab.), arXiv:1803.05656
A lot of previous studies

Color electric field
Cea et al., PRD88 (2012) 054504.

Action density
Cardoso et al., PRD86 (2013) 054501.
A lot of previous studies

Color electric field
Cea et al., PRD88 (2012) 054504.

Action density
Cardoso et al., PRD86 (2013) 054501.

More direct physical quantity: Stress tensor!!
Measurement on the lattice

To do:

① Prepare $Q \bar{Q}$ on the lattice  
② Measure EMT around $Q \bar{Q}$
Wilson Loop

\[ \langle W(R, T) \rangle = C_0 \exp[-V(R)T] + C_1 \exp[-V_1(R)T] + \cdots \]

\[ V(R) = -\lim_{T \to \infty} \frac{1}{T} \log \langle W(R, T) \rangle \]

Ground state potential

Confinement potential

\[ \beta = 6.600 \ (a = 0.038 \text{ fm}) \]

Measurement on the lattice

To do

① Prepare \( Q\bar{Q} \) on the lattice ② Measure EMT around \( Q\bar{Q} \)

quenched SU(3) Yang-Mills
**Measurement on the lattice**

**To do**

1. Prepare $Q\bar{Q}$ on the lattice
2. Measure EMT around $Q\bar{Q}$

**Gradient flow**

Flow eq. (Lüscher, 2010)

$$\frac{\partial B_\mu(t,x)}{\partial t} = -g_0^2 \frac{\delta S[B]}{\delta B_\mu(t,x)}$$

$B_\mu$ : smeared field

**EMT defined via gradient flow** (Suzuki, 2013)

$$T_{\mu\nu}(t,x) = \frac{1}{\alpha_u(t)} U_{\mu\nu}(t,x) + \frac{\delta_{\mu\nu}}{4\alpha_E(t)} [E(t,x) - \langle E(t,x) \rangle] + O(t)$$

**Entropy density vs. temperature**

FlowQCD (2016)

Integral method

- Ref. [1]
- Ref. [4]

**FlowQCD (2016)**

**Gradient flow**

**Entropy density vs. temperature**

**Measurement on the lattice**

1. Prepare $Q\bar{Q}$ on the lattice
2. Measure EMT around $Q\bar{Q}$

**Gradient flow**

Flow eq. (Lüscher, 2010)

$$\frac{\partial B_\mu(t,x)}{\partial t} = -g_0^2 \frac{\delta S[B]}{\delta B_\mu(t,x)}$$

$B_\mu$ : smeared field

**EMT defined via gradient flow** (Suzuki, 2013)

$$T_{\mu\nu}(t,x) = \frac{1}{\alpha_u(t)} U_{\mu\nu}(t,x) + \frac{\delta_{\mu\nu}}{4\alpha_E(t)} [E(t,x) - \langle E(t,x) \rangle] + O(t)$$

**Entropy density vs. temperature**

FlowQCD (2016)

Integral method

- Ref. [1]
- Ref. [4]
Setup

- Quenched SU(3) Yang-Mills gauge theory
- Wilson gauge action
- Clover operator
- Continuum limit
- APE smearing for spatial links
- Multihit improvement in temporal links
- Simulation using BlueGene/Q @ KEK

<table>
<thead>
<tr>
<th>$\beta$</th>
<th>Lattice spacing</th>
<th>Lattice size</th>
<th># of statistics</th>
</tr>
</thead>
<tbody>
<tr>
<td>6.304</td>
<td>0.057 fm</td>
<td>$48^4$</td>
<td>140</td>
</tr>
<tr>
<td>6.465</td>
<td>0.046 fm</td>
<td>$48^4$</td>
<td>440</td>
</tr>
<tr>
<td>6.513</td>
<td>0.043 fm</td>
<td>$48^4$</td>
<td>600</td>
</tr>
<tr>
<td>6.600</td>
<td>0.038 fm</td>
<td>$48^4$</td>
<td>1500</td>
</tr>
<tr>
<td>6.819</td>
<td>0.029 fm</td>
<td>$64^4$</td>
<td>1000</td>
</tr>
</tbody>
</table>
Stress distribution — Maxwell theory — (revisit)

\[ T_{ij} = \epsilon_0 \left( E_i E_j - \frac{\delta_{ij}}{2} E^2 \right) + \frac{1}{\mu_0} \left( B_i B_j - \frac{\delta_{ij}}{2} B^2 \right) \]

- Perpendicular plane: \( \lambda_k < 0 \)
- Parallel plane: \( \lambda_k > 0 \)

\[ T_{ij} n_j^{(k)} = \lambda_k n_i^{(k)} \]

\((i, j = 1, 2, 3; k = 1, 2, 3)\)

Length of arrows = \( \sqrt{|\lambda_k|} \)
Stress distribution — SU(3) YM theory —

\[ a = 0.029 \text{ fm (no continuum limit)} \]
\[ t/a^2 = 2.0 \text{ (no } t \rightarrow 0 \text{ limit)} \]
\[ R = 0.69 \text{ fm} \]
\[ \text{Length of arrows} = \sqrt{|\lambda_k|} \]
Stress distribution — SU(3) YM vs. Maxwell —

SU(3) YM theory

Maxwell theory

✓ Gauge Invariant
✓ Local interaction
✓ Propagation of force: squeezed vs. spreading
We will focus on the mid-plane: double extrapolation
Double extrapolation @ mid point

\[ O_{\text{lat}} = O_{\text{cont}} + A_0 t + A_1(t) a^2 + \cdots \]

\[ \langle \overline{\mathcal{T}}_{22}(t, 0) \rangle_{Q\overline{Q}} [\text{GeV/fm}^3] \]

\[ t/a_{6.819} \]

\[ a = 0.029 \text{ fm} \]
\[ a = 0.046 \text{ fm} \]
\[ a = 0.038 \text{ fm} \]
\[ a = 0.058 \text{ fm} \]
Double extrapolation @ mid point

\( O_{\text{lat}} = O_{\text{cont}} + A_0 t + A_1(t) a^2 + \ldots \)

\( a \to 0 \) limit
Double extrapolation @ mid point

FlowQCD (2016)

\[
\langle T_{zz}^{\text{lat}}(t, 0) \rangle_{Q\bar{Q}} [\text{GeV}/\text{fm}^3] = \text{continuum}
\]

\[
O_{\text{lat}} = O_{\text{cont}} + A_0 t + A_1 a^2 + \ldots
\]
Double extrapolation @ mid point

\[ O_{\text{lat}} = O_{\text{cont}} + A_0 t + A_1 a^2 + \ldots \]

FlowQCD (2016)
Cylindrical coordinate

\[
T_{\mu\nu} = \begin{pmatrix}
T_{44} & 0 & 0 & 0 \\
0 & T_{zz} & 0 & 0 \\
0 & 0 & T_{rr} & 0 \\
0 & 0 & 0 & T_{\theta\theta}
\end{pmatrix}
\]

Diagonalization of EMT (cylindrical and parity symmetry)

Degeneracy in Maxwell theory

\[-T_{44} = -T_{zz} = T_{rr} = T_{\theta\theta}\]
EMT distribution in mid plane

$R = 0.46 \text{ fm}$

$\langle T_{rr}(r) \rangle_{\bar{Q}Q}$

$\langle T_{zz}(r) \rangle_{\bar{Q}Q}$

$\langle T_{\theta\theta}(r) \rangle_{\bar{Q}Q}$

$\langle T_{44}(r) \rangle_{Q\bar{Q}}$

$\langle T_{zz}(r) \rangle_{Q\bar{Q}}$

$\langle T_{rr}(r) \rangle_{Q\bar{Q}}$

$\langle T_{\theta\theta}(r) \rangle_{Q\bar{Q}}$

$V(R) [\text{GeV}]$

$r [\text{fm}]$

$R [\text{fm}]$

Talk @ ELPH (2018/09/12)
EMT distribution in mid plane

(a) $R = 0.46 \text{ fm}$

(b) $R = 0.69 \text{ fm}$

(c) $R = 0.92 \text{ fm}$

- $\langle T_{44}^{R}(r) \rangle_{Q\bar{Q}} \text{ [GeV/fm}^3\text{]}$
- $\langle T_{zz}^{R}(r) \rangle_{Q\bar{Q}} \text{ [GeV/fm}^3\text{]}$
- $\langle T_{rr}^{R}(r) \rangle_{Q\bar{Q}} \text{ [GeV/fm}^3\text{]}$
- $\langle T_{66}^{R}(r) \rangle_{Q\bar{Q}} \text{ [GeV/fm}^3\text{]}$
EMT distribution in mid plane

Properties in non-Abelian theory

- $T_{44} \approx T_{zz}, T_{rr} \approx T_{\theta\theta}$ (Degeneracy)
- $T_{44} \neq T_{rr}$ (Separation)
- $\sum_\mu T_{\mu\mu} \neq 0$ (non-zero trace anomaly)

![Diagram showing EMT distribution in mid plane](image)

(a) $R = 0.46 \text{ fm}$  
(b) $R = 0.69 \text{ fm}$  
(c) $R = 0.92 \text{ fm}$

- $\langle T_{44}^R(r) \rangle_{Q\bar{Q}} [\text{GeV/fm}^3]$
- $\langle T_{zz}^R(r) \rangle_{Q\bar{Q}} [\text{GeV/fm}^3]$
- $\langle T_{rr}^R(r) \rangle_{Q\bar{Q}} [\text{GeV/fm}^3]$
- $\langle T_{\theta\theta}^R(r) \rangle_{Q\bar{Q}} [\text{GeV/fm}^3]$
Local interaction vs. action at a distance

EMT (local interaction)

confinement potential
(action at a distance)

\[ V(R) = a + bR + c/R \]

\[ F_{\text{stress}} := \int_{\text{mid}} \langle T_{zz} \rangle_{Q\bar{Q}} d^2x \]

\[ F_{\text{pot}} := -\frac{dV(R)}{dR} \]
Local interaction vs. action at a distance

EMT (local interaction)

action at a distance

confinement potential

$\rho \equiv -F_{\text{stress}}$

$V(R) = a + bR + c/R$

$F_{\text{stress}} := \int_{\text{mid}} \langle T_{zz} \rangle_{Q\bar{Q}} \, d^2x$

$F_{\text{pot}} := -\frac{dV(R)}{dR}$

Good agreement!!
Goal and Method

**Goal**

Stress distribution in $Q\bar{Q}$ system in terms of local interaction

**Method**

Gradient Flow

Lattice study

RY+ (FlowQCD collab.), arXiv:1803.05656

Model Analysis

RY, Takumi Iritani and Masakiyo Kitazawa in progress
Stress tensor distribution around static $\bar{Q}Q$ via Dual Abelian–Higgs model

RY, Takumi Iritani and Masakiyo Kitazawa

in progress
\[ \mathcal{L}_{DAH} = -\frac{1}{4} (\partial_{\mu} B_{\nu} - \partial_{\nu} B_{\mu})^2 + \left| (\partial_{\mu} + ig B_{\mu}) \phi \right|^2 - \lambda (\phi^2 - v^2)^2 \]

- DAH has solution describing flux tube

- Symmetry-broken vacuum and recovery of symmetry

- “Trace anomaly” \((\sum_{\mu} T_{\mu\mu} \neq 0)\)

- Phenomenological model in QCD
  (Dual Ginzburg-Landau model)
Dual Abelian-Higgs (DAH) Model

\[ \mathcal{L}_{DAH} = -\frac{1}{4} (\partial_\mu B_\nu - \partial_\nu B_\mu)^2 + |(\partial_\mu + igB_\mu)\phi|^2 - \lambda(\phi^2 - v^2)^2 \]

✓ DAH has solution describing flux tube

✓ Symmetry-broken vacuum and recovery of symmetry

✓ “Trace anomaly” \((\sum_\mu T_{\mu\mu} \neq 0)\)

✓ Phenomenological model in QCD
   (Dual Ginzburg-Landau model)

To Do

✓ Solve DAH with the approximation of translational symmetry
✓ Can DAH model explain the lattice results??
EMT distribution in infinitely-long flux tube

GL parameter $\kappa = \sqrt{\lambda} / g$ :

$\kappa < 1 / \sqrt{2} \ (I)$, $\kappa > 1 / \sqrt{2} \ (II)$

$\kappa = 1 / \sqrt{2}$

$T_{44}(r) = T_{zz}(r)$ (degeneracy at any $\kappa$)

No transverse force in flux tube (Bogomol’nyi bound)

de Vega et al., PRD 14, 1100 (1976).
EMT distribution in infinitely-long flux tube

κ = 0.20 (type I)

κ = 2.0 (type II)

- Sign of $T_{rr}(r)$: Positive (type I), Negative (type II)
- Conservative law $\frac{\partial}{\partial r} (rT_{rr}) = T_{\theta\theta}$: $T_{\theta\theta}$ can change sign
- Separation b/w $T_{rr}, T_{\theta\theta}$ (Conservative law)
Lattice study vs. DAH model analysis

**Lattice study**

- $R = 0.92 \text{ fm}$

- Plot showing $\langle T_{44}^R(r) \rangle_{Q\bar{Q}}$, $\langle T_{zz}^R(r) \rangle_{Q\bar{Q}}$, $\langle T_{rr}^R(r) \rangle_{Q\bar{Q}}$, and $\langle T_{\theta\theta}^R(r) \rangle_{Q\bar{Q}}$ versus $r$ [fm].

**DAH model analysis (type I)**

- $-T_{44}(r)$, $-T_{zz}(r)$, $T_{rr}(r)$, $T_{\theta\theta}(r)$

**Inconsistency b/w two analyses $\Rightarrow$ Finite length effect**
EMT distribution in finite-length flux tube

\( \kappa = 0.60 \) (type I )

Preliminary

\[ R = 0.91 \, \text{fm}, \xi_B = 0.22 \, \text{fm}, \xi_\phi = 0.26 \, \text{fm} \]

Finite length effect of the flux tube is crucial !!
Summary

✓ First measurement of stress distribution on the lattice !!
✓ Stress distribution in $Q\bar{Q}$ system via DAH model

Outlook

✓ Validity and limitation of DAH model
✓ EMT distribution inside hadrons, finite temperature, full QCD...
Back up
Ground state saturation

\[ -\langle T_{zz}(t,0) \rangle_{Q\bar{Q}}^{\text{lat}} \text{ [GeV/fm}^3\text{]} \]

\[ \text{vs} \ T/\bar{\alpha} \]

- $t/\bar{\alpha}^2 = 1.0$
- $t/\bar{\alpha}^2 = 1.7$
- $t/\bar{\alpha}^2 = 2.0$
- $t/\bar{\alpha}^2 = 3.0$
- $t/\bar{\alpha}^2 = 4.0$
- $t/\bar{\alpha}^2 = 6.0$